



Calculations Policy 2022-23

Document Status			
Date of Next Review		Responsibility	<i>Full Governing Body</i>
Success Criteria for review completion		Responsibility	<i>Headteacher/Chair of Governors</i>
Date of Policy Creation	June 2022	Responsibility	<i>Chair of Governors</i>
Date of Policy Adoption by Governing Body	Signed: Headteacher Date: Chair of Governors Date:		
Method of Communication: School Website			

Statement of Intent

At North Duffield Primary School, we work with relentless consistency to enable all our children to achieve our school vision, "Growing Learners for Life," Every child will make good or better progress through the high-quality teaching they will receive, and their engagement with our diverse, absorbing and exciting curriculum which provides appropriate and relevant enrichment experiences.

We model and promote an inclusive ethos so our learners are flexible, imaginative, responsible and confident life-long learners, who value "Rights, Respect, Responsibility and Independence." The fundamental British values of democracy, the rule of law, individual liberty, mutual respect and tolerance of those of different faiths and beliefs, are promoted throughout our school curriculum, and our whole school ethos. This ensures that all children including those with SEND or medical conditions have the same opportunities and expectations placed upon them as all other young people.

Everyone at North Duffield Primary School has the right to feel welcome, secure and happy. It is the responsibility of the Governing Body and Headteacher to ensure that all members of the school community work within a safe and enabling environment. Our policies have been developed in line with the school's vision, aims and values.

This calculations policy reflects the intent of mathematics within the National Curriculum for England and Wales 2013 and how at North Duffield Primary School we will teach calculations. It states:

‘The National Curriculum for mathematics aims to ensure that all pupils become fluent in the fundamentals of mathematics... reason mathematically.... and can solve problems....’

‘Mathematics is an interconnected subject...Pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.’

Calculations in Early Years Foundation Stage

Mathematics – Number- addition and subtraction

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Calculations within Milestone One:

Using number facts	Recall and use addition and subtraction facts to 20 fluently (Y1), and derive and use related facts up to 100 (Y2).
Complexity	Apply increasing knowledge of mental and written methods. Add and subtract one-digit and two-digit numbers to 20 (Y1), and recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers (Y2).

Calculations within Milestone Two:

Using number facts	Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction	
Complexity	Solve two-step addition and subtraction problems in contexts, deciding which operations and methods to use and why.	
	<table border="1"><tr><td>Y3 Add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds. Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Solve problems including missing number problems using number facts, place value and more complex addition and subtraction.</td><td>Y4 Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Solve addition and subtraction two-step problems in contexts, deciding which operations to use and why.</td></tr></table>	Y3 Add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds. Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Solve problems including missing number problems using number facts, place value and more complex addition and subtraction.
Y3 Add and subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds. Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Solve problems including missing number problems using number facts, place value and more complex addition and subtraction.	Y4 Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Solve addition and subtraction two-step problems in contexts, deciding which operations to use and why.	

Calculations within Milestone Three:

Methods	Add and subtract whole numbers with more than 4 digits, including using formal written methods. (columnar addition and subtraction)	
	Add and subtract numbers mentally with increasingly large numbers	
Complexity	Solve multi-step addition and subtraction problems in contexts, deciding which operations and methods to use and why.	
	Y5 Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction). Solve addition and subtraction multi-step problems in contexts, deciding which operations to use and why.	Y6 To undertake mental calculations with increasingly large numbers and more complex calculations. Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

At North Duffield Primary School, we follow the Chris Quigley Essentials for Learning philosophy whilst using a very **concrete - pictorial – abstract** approach to teaching numeracy. We show and use concrete examples with the children to make it ‘real’ to them, before moving onto using pictorial representations of these. Finally, we move onto the abstract representation of the concepts. We can revisit any of the stages at any point, should we feel that an individual child or group of children needs reinforcement or further practice.






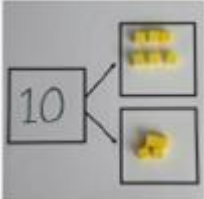
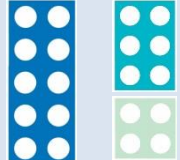

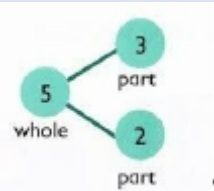

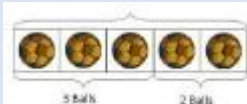
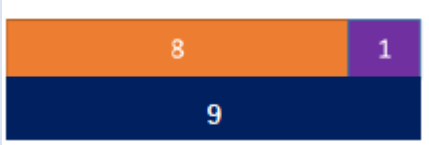
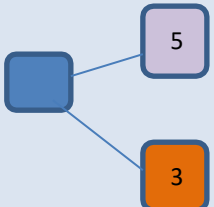
The following pages give an insight to the progression across all year groups and milestones at our school with examples of how **concrete - pictorial – abstract** can be used to teach calculations.


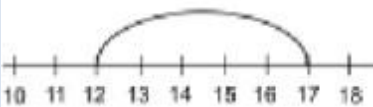

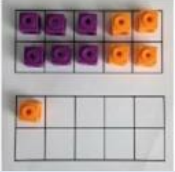
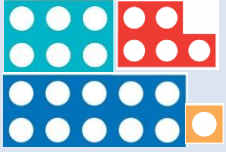
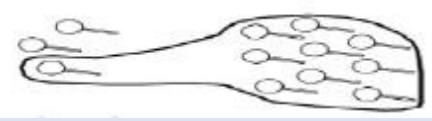
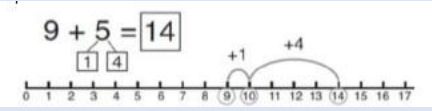
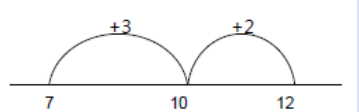
Page 5	Progression in calculations across year groups Reception – Y6
Pages 6-9	Addition
Pages 10-14	Subtraction
Pages 15-20	Multiplication
Pages 21-27	Division


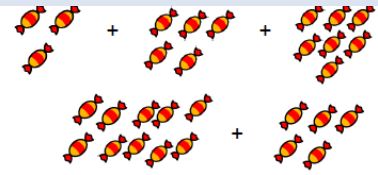
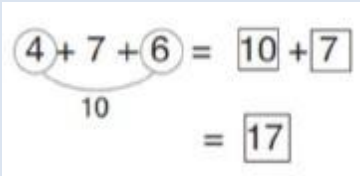
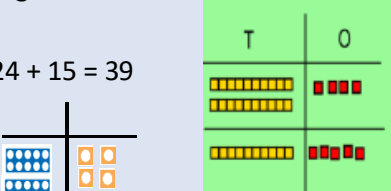
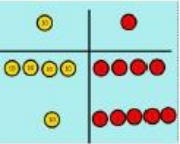
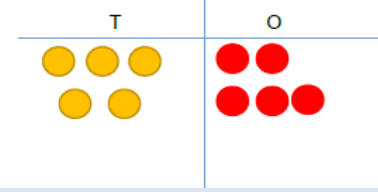
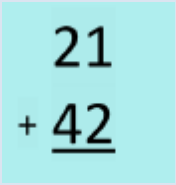
Progression in each calculation

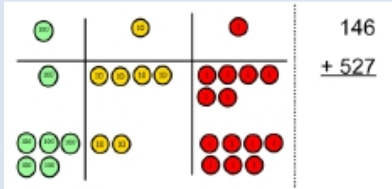
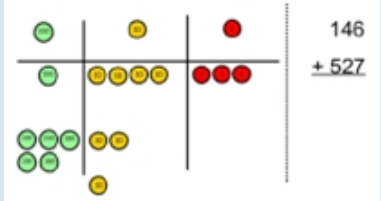
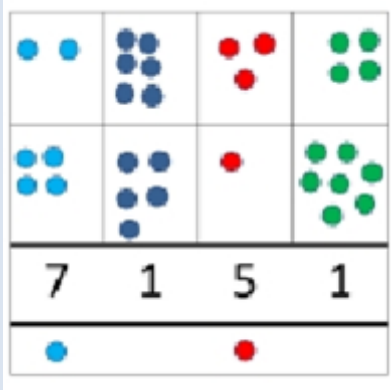
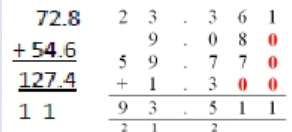
	Year R	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Counting a set of objects. Knowing 1 more or 1 less Place numbers in order of size	Combining two parts to make a whole: part whole model Starting at the bigger number and counting on Regrouping to make 10	Adding three single digits. Column method – no regrouping	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	One less than / Taking away ones	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4-digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2-digit number)
Division	Halving	Sharing objects equally Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1-digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2-digit number- interpret remainders as whole numbers, fractions or round)

Addition


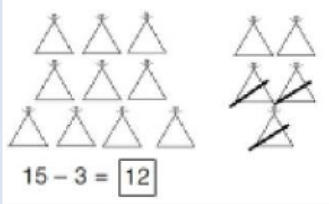



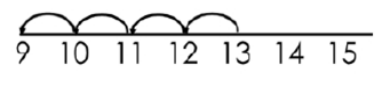
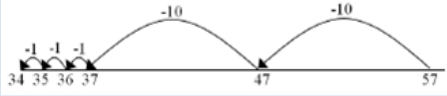
Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract										
<p>Stage 1</p> <p>Yr R</p>	<p>Counting a set of objects</p> <p>Knowing 1 more or 1 less</p> <p>Place numbers in order of size</p>	<p>One more</p> <p>One less</p> <p>Bigger</p> <p>Larger</p>			<table border="1" data-bbox="1691 327 1915 566"> <tr> <td></td> <td>3</td> </tr> <tr> <td></td> <td>5</td> </tr> <tr> <td></td> <td>1</td> </tr> <tr> <td></td> <td>2</td> </tr> <tr> <td></td> <td>4</td> </tr> </table>		3		5		1		2		4
	3														
	5														
	1														
	2														
	4														
<p>Stage 2</p> <p>Yr R + 1</p>	<p>Combining two parts to make a whole: part-whole model</p>	<p>Addition</p> <p>Sum</p> <p>Total</p> <p>Parts and wholes</p> <p>Plus</p> <p>Add</p> <p>Altogether</p> <p>More than</p> <p>Equal to</p> <p>Same as</p>	<p>Use cubes to add two numbers together as a group or in a bar:</p>      	<p>Use pictures to add two numbers together as a group or in a bar:</p>    	<p>Use the part-part whole diagram as shown to move into the abstract:</p>  <p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>										


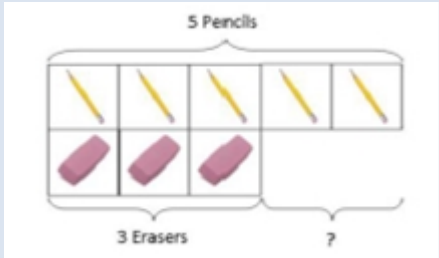
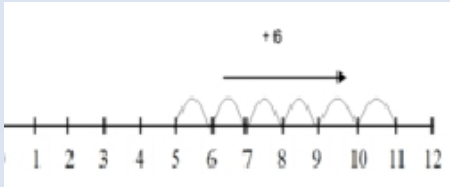
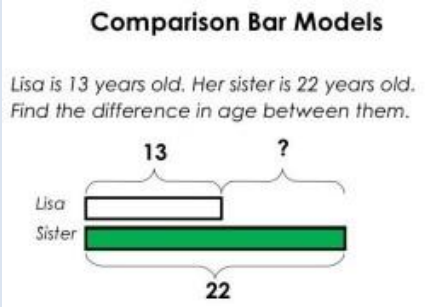
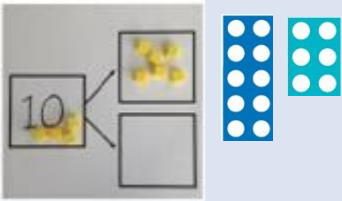
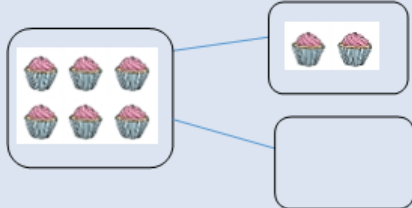
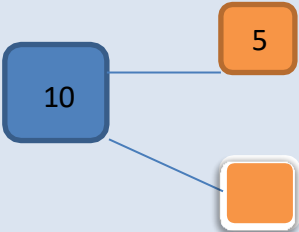
Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 3</p> <p>Yr 1</p>	<p>Start at the bigger number and count on</p>		 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p> <p>$12 + 5 = 7$</p> 	<p>Place the larger number in your head and count on the smaller number to find your answer.</p> <p>$5 + 12 = 17$</p>
<p>Stage 4</p> <p>Yr 1</p>	<p>Regrouping to make 10</p>	<p>Regroup</p> <p>Partition</p>	<p>Regroup $9 + 3$ into $10 + 2$ before adding together:</p>  <p>Start with the larger number and use the smaller number to make 10</p>  <p>$6 + 5 = 11$</p> 	<p>Use pictures or a number line. Regroup or partition the smaller number to make 10 before adding.</p> <p>$3 + 9 =$</p>   <p>Children move on to using an 'empty number line'.</p> <p>E.g. $7 + 5$ becomes $7 + 3 + 2$</p> 	<p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10? How many more do I add on now?</p> <p>$7 + 5 = 7 + 3 + 2 = 12$</p>


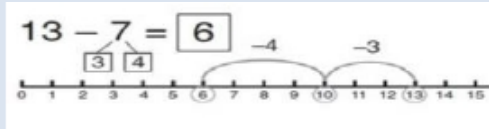
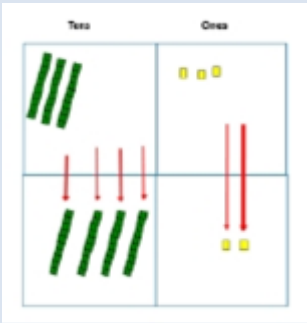
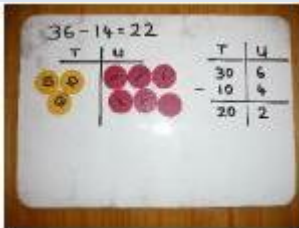

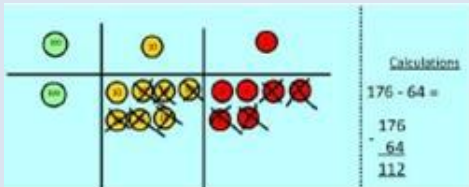
Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p><u>Stage</u> 5</p> <p>Yr 2</p>	<p>Adding three single digits</p>	<p>Addition Sum Total Parts and wholes Plus Add Altogether More than Equal to Same as</p>	<p>$4 + 7 + 6 = 17$ Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>	 <p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p>	 <p>Combine the two numbers that make 10 and then add on the remainder.</p>
<p><u>Stage</u> 6</p> <p>Yr 2</p>	<p>Column addition – without regrouping</p>	<p>Regroup Partition</p>	<p>Partition the numbers into tens and ones using base 10 blocks, place value counters.</p> <p>Add together the ones first then add the tens. Finally add the 2 totals together.</p> <p>$24 + 15 = 39$</p>  <p>$44 + 15 = 59$</p> 	<p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p> <p>$32 + 23 = 55$</p> 	<p>$21 + 42 =$</p>  <p>Record the calculation vertically adding the column of ones then the column of tens.</p>

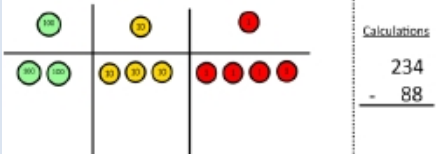
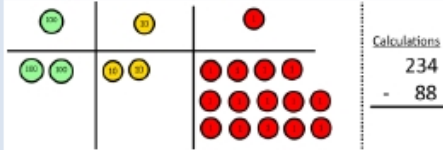
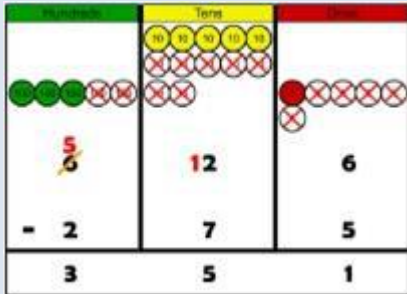



Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 7</p> <p>Yr 3 (3 digits)</p> <p>Yr 4 (4 digits)</p> <p>Yr 5 + (4+ digits and decimals with same np. dp)</p> <p>Yr 6 (decimals with no. dp)</p>	<p>Column addition – with regrouping</p>	<p>Exchange</p> <p>Regroup</p> <p>Partition</p>	<p>Make both numbers with place value counters.</p>  <p>Add up the ones and exchange 10 ones for one 10. Add it to the other tens:</p>  <p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.</p> <p>This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.</p> <p>As children move on to decimals, money and decimal place value counters can be used to support learning.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p> 	<p>Begin by partitioning the numbers:</p> <p style="text-align: center;">For 76 + 47</p> $\begin{array}{r} 70 + 6 \\ 40 + 7 \\ \hline 110 + 13 = 123 \end{array}$ <p>Move on to clearly show the exchange below the addition:</p> $\begin{array}{r} 70 + 6 \\ 40 + 7 \\ \hline 120 + 3 = 123 \\ \color{red}{10} \end{array}$ <p>This then becomes the compact method where numbers aren't partitioned but exchanges still take place:</p> $\begin{array}{r} 76 \\ + 47 \\ \hline 123 \\ \color{red}{11} \end{array}$ <p>As the children move on, introduce decimals with and without the same number of decimal places. Money can also be used here.</p> 

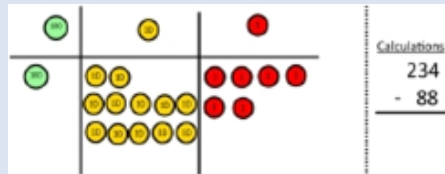
Subtraction

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
Stage 1 Yr R Yr 1	One less than / Taking away ones	One less Take away Less than The difference Subtract Minus Fewer Decrease	Use physical objects, counters, cubes numicon, etc, to show how objects can be taken away. $6 - 2 = 4$ 	Cross out drawn objects to show what has been taken away.  	$18 - 3 = 15$ $8 - 2 = 6$ Although number sentences are recorded in the concrete and pictorial methods children are introduced to them on their own while encouraging them to mentally take away ones.
Stage 2 Yr R Yr 1 Yr 2	Counting back	One less Take away Less than The difference Subtract Minus Fewer Decrease	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. $13 - 4$  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2-digit numbers.	For $13 - 4$, put 13 in your head and count back 4. What number are you at? Use your fingers to help.

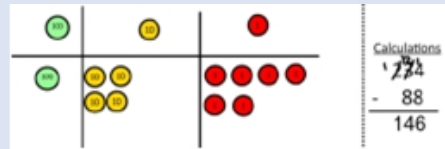
Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 3</p> <p>Yr 1</p> <p>Yr 2</p>	<p>Find the difference</p>	<p>One less</p> <p>Take away</p> <p>Less than</p> <p>The difference</p> <p>Subtract</p> <p>Minus</p> <p>Fewer</p> <p>Decrease</p>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference.</p>  <p>Use basic bar models with items to find the difference.</p>	<p>Count on to find the difference:</p>  <p>Draw bars to find the difference between 2 numbers.</p> 	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
<p>Stage 4</p> <p>Yr 1</p> <p>Yr 2</p>	<p>Part Whole Model</p>	<p>Part</p> <p>Whole</p> <p>Inverse</p>	<p>Link to addition - use the part whole model to help explain the inverse between addition and subtraction.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> $10 - 6 =$	<p>Use a pictorial representation of objects to show the part whole model.</p>  $6 - 2 = 4$	 <p>Move to using numbers within the part whole model.</p>

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract																												
Stage <u>5</u> Yr 1 Yr 2	Make 10	Ten frame Remaining Take off Count back	$14 - 5 =$  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	Start at 13. Count back 3 to reach 10. Then count back the remaining 4 so you have taken away 7 altogether. You have reached your answer. 	$16 - 8 =$ How many do we take off to reach the previous 10? (6) How many do we have left to take off? (2)																												
Stage <u>6</u> Yr 2	Column method without regrouping	Column Partition Larger	$75 - 42$  Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again, make the larger number first. 	Draw the Base 10 or place value counters alongside the written calculation to help to show working:  	Partitioned numbers are written vertically: For $54 - 22$ <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>Tens</td> <td>Ones</td> <td></td> </tr> <tr> <td></td> <td>50</td> <td>4</td> <td></td> </tr> <tr> <td>-</td> <td>20</td> <td>2</td> <td></td> </tr> <tr> <td></td> <td colspan="2"><hr/></td> <td></td> </tr> <tr> <td></td> <td>30</td> <td>+ 2</td> <td>= 32</td> </tr> </table> This will lead to a clear written column subtraction: <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td>32</td> </tr> <tr> <td>-</td> <td>12</td> </tr> <tr> <td></td> <td><hr/></td> </tr> <tr> <td></td> <td>20</td> </tr> </table>		Tens	Ones			50	4		-	20	2			<hr/>				30	+ 2	= 32		32	-	12		<hr/>		20
	Tens	Ones																															
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Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 7</p> <p>Yr 3 (3 digits)</p> <p>Yr 4 (4 digits)</p> <p>Yr 5 + (4+ digits and decimals with same np. dp)</p> <p>Yr 6 (decimals with diff no. dp)</p>	<p>Column method with regrouping</p>	<p>Exchange</p> <p>Partition</p>	<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters.</p>  <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ <p>Start with the ones, can I take away 8 from 4 easily? I can't take away 8 ones. I need to exchange one of my tens for ten ones.</p>  <p>Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$ <p>Now I can subtract 8 ones from 14. Next look at the tens. I can't take away 8 tens. I need to exchange a hundred for 10 tens:</p>	<p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make:</p>  <p>When confident, children can find their own way to record the exchange/regrouping:</p>  <p>Just writing the numbers as shown here shows that the child</p>	 <p>Children can start their formal written method by partitioning the number into clear place value columns.</p>  <p>Moving forward the children use a more compact method.</p> <p>This will lead to an understanding of subtracting any number including decimals:</p>

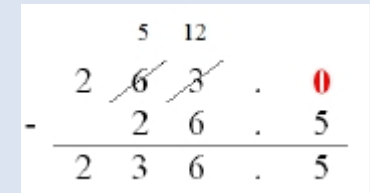


Now I can take eight tens from the 12 tens and complete the subtraction.

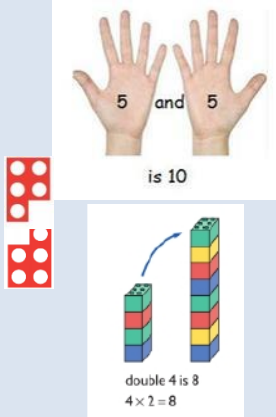
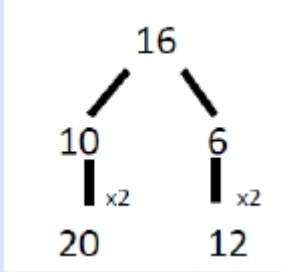

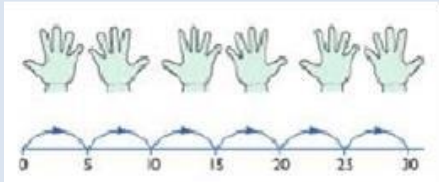


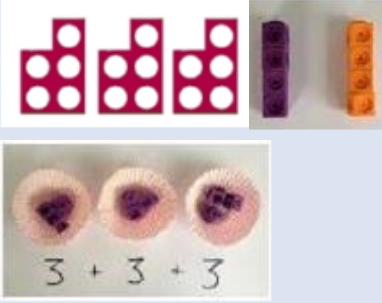

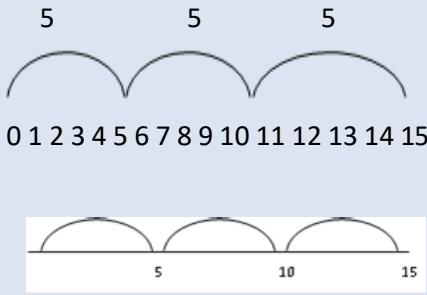

Show children how the concrete method links to the written method alongside their working. Cross out the numbers when exchanging and show where we write our new amount.


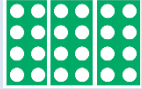

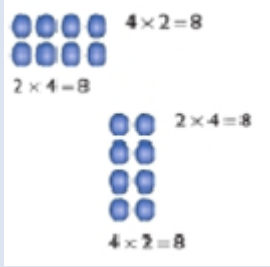
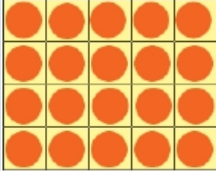

understands the method and knows when to exchange/regroup.

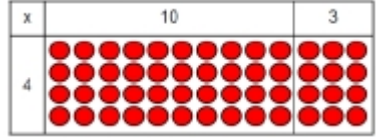
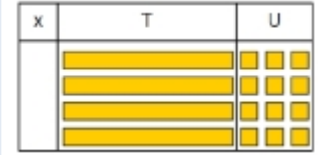

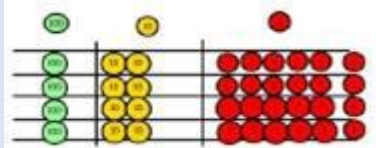
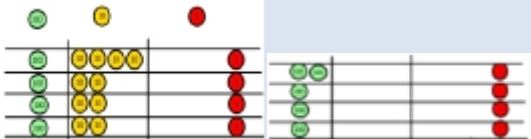
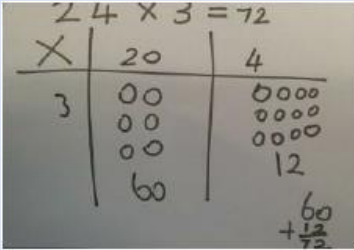


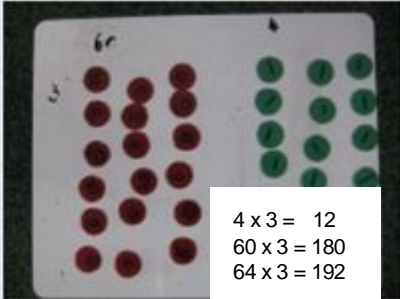
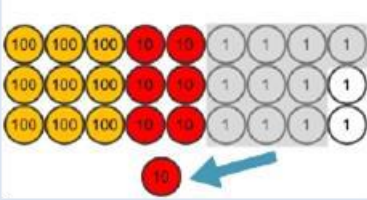
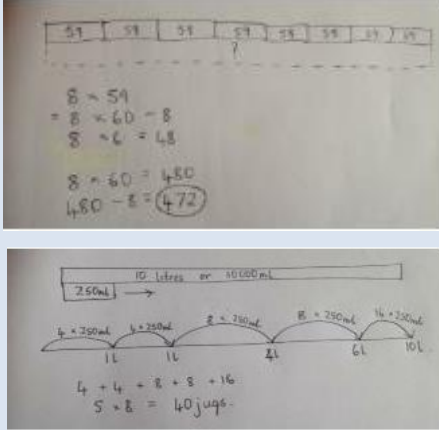
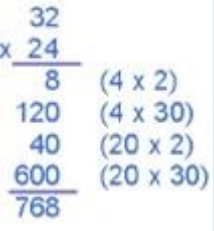
Multiplication

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage <u>1</u></p> <p>Yr R</p> <p>Yr 1</p> <p>Yr 2</p>	<p>Doubling</p>	<p>Double</p> <p>Count on (from, to)</p> <p>Count back (from, to)</p> <p>Count in ones, twos, tens...</p> <p>Is the same as</p>	<p>Use practical activities to show how to double a number.</p> 	<p>Draw pictures to show how to double a number.</p>	 <p>Partition a number and then double each part before recombining it back together.</p> $4 \times 2 = 8$
<p>Stage <u>2</u></p> <p>Yr R</p> <p>Yr 1 + Yr 2 (x2, 5, 10)</p> <p>Yr3 (x3, 4, 8)</p>	<p>Counting in multiples</p>	<p>Multiplied by</p> <p>The product of</p> <p>Groups of</p> <p>Lots of</p> <p>Is equal to</p>	<p>Count in multiples supported by concrete objects in equal groups.</p> 	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count out loud in multiples of a number.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p><u>Stage</u> 3</p> <p>Yr 2</p> <p>Yr 3</p>	<p>Repeated addition</p>		<p>Use different objects to add equal groups.</p> 	<p>There are 3 plates. Each plate has 2-star biscuits on. How many biscuits are there?</p>  <p>Repeated addition can be shown on a labelled or empty number line.</p> <p>Eg $5 + 5 + 5 = 15$:</p>  <p>Begin to relate repeated addition to multiplication using 'lots of'.</p> <p>e.g. 3 lots of 5 = 15</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>This then leads to writing related multiplication sentences e.g. $2 \times 5 = 10$</p>

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p><u>Stage</u> 4</p> <p>(Yr 1)</p> <p>Yr 2</p> <p>Yr 3</p>	<p>Arrays - showing commutative multiplication</p>	<p>Array</p> <p>Commutative</p>	<p>Create arrays using counters /cubes /numicon to show multiplication sentences.</p> <p>Eg $4 \times 6 = 24$</p>   <p>Begin to look at arrays in different orientations to make the link between.</p>  <p>Eg $5 \times 3 = 15$ and $3 \times 5 = 15$ (commutativity)</p>	<p>Draw arrays in different rotations to find commutative multiplication sentences.</p>  <p>Link arrays to area of rectangles:</p> 	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$</p> <p>$3 + 3 + 3 + 3 + 3 = 15$</p> <p>$5 \times 3 = 15$</p> <p>$3 \times 5 = 15$</p>

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract																																							
<p><u>Stage</u> <u>5</u> Yr 3</p>	<p>Grid Method</p>	<p>Grid Exchange</p>	<p>Show the link with arrays to first introduce the grid method. Eg 4×13 4 rows of 10; 4 rows of 3</p>  <p>Move on to using Base 10 to move towards a more compact method:</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. Here, we are multiplying 126 by 4 so we need 4 rows with each containing 26.</p>  <p>Fill each row with 126.</p>  <p>Add up each column, starting with the ones making any exchanges needed. Then you have your answer.</p> 	<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below:</p> 	<p>Start with multiplying 2-digit by 1-digit numbers and showing the addition alongside the grid:</p> <table border="1" data-bbox="1783 416 2033 544"> <tr><td>x</td><td>30</td><td>5</td></tr> <tr><td>7</td><td>210</td><td>35</td></tr> <tr><td colspan="3">$210 + 35 = 245$</td></tr> </table> <p>Moving forward, multiply by 2, 3 and 4-digit numbers showing the different rows within the grid method:</p> <p>13×28</p> <table border="1" data-bbox="1688 711 1944 834"> <tr><td>x</td><td>20</td><td>8</td></tr> <tr><td>10</td><td>200</td><td>80</td></tr> <tr><td>3</td><td>60</td><td>24</td></tr> </table> <table border="1" data-bbox="1951 746 2072 911"> <tr><td>280</td></tr> <tr><td>+ 84</td></tr> <tr><td><hr/></td></tr> <tr><td>364</td></tr> <tr><td><hr/></td></tr> <tr><td>1</td></tr> </table> <p>Eg 1342×18</p> <table border="1" data-bbox="1738 946 2074 1106"> <tr><td>x</td><td>1000</td><td>300</td><td>40</td><td>2</td></tr> <tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr> <tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr> </table>	x	30	5	7	210	35	$210 + 35 = 245$			x	20	8	10	200	80	3	60	24	280	+ 84	<hr/>	364	<hr/>	1	x	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
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Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 6</p> <p>Yr4 (2 and 3-digit x 1 digit)</p> <p>Yr 5 (4 digits x 1 or 2 digits)</p> <p>Yr 6 (4 digits x 2 digits)</p>	<p>Column multiplication</p>	<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters for carrying out column multiplication. They can partition and record each calculation vertically.</p>  <p>It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.</p> <p>The idea of exchanging will support them in moving on to a more compact method: Eg 3 x 324</p> 	<p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>  <p>As with stage 4, children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.</p>	<p>Start with long multiplication, reminding the children about lining up their numbers clearly in columns. As with the grid method, numbers of more than one digit are partitioned but this time the calculation is recorded vertically. To support them, children need to write out what they are solving next to their answer.</p> <p>Eg 38 x 7</p> $\begin{array}{r} 38 \\ \times 7 \\ \hline 56 \quad (8 \times 7) \\ 210 \quad (30 \times 7) \\ \hline 266 \end{array}$  <p>This moves to the more compact method, examples shown overleaf.</p>

Start by multiplying the ones digit, recording the last digit of the answer in the answer line but exchanging any tens and putting them under the tens column to be added on after multiplying the tens digit. Again, the last digit in the answer is recorded in the answer line and any hundred are exchanged, this time to the hundred's column, and so on.

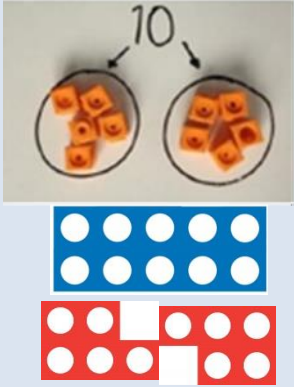
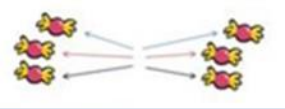




Eg 38×7


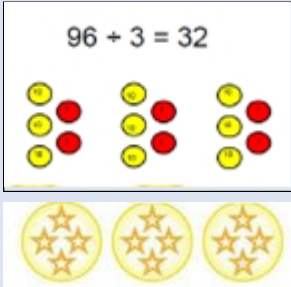

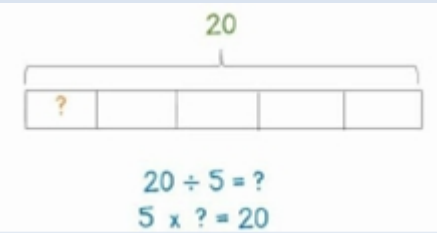

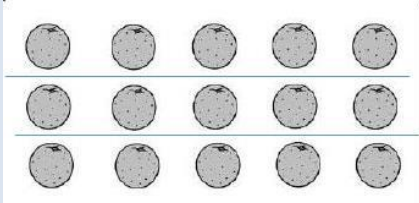
$$\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ 5 \end{array}$$

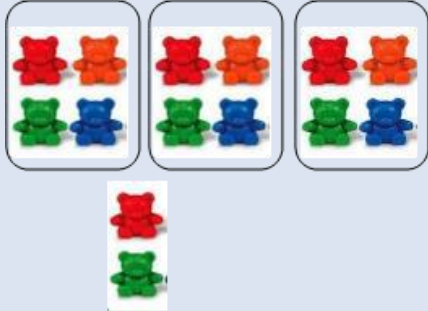

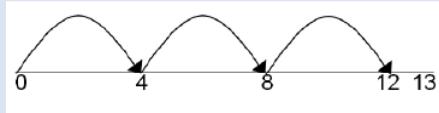
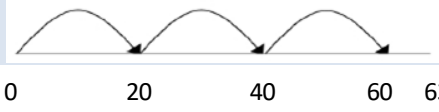
Eg 38×27


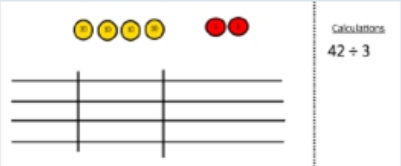

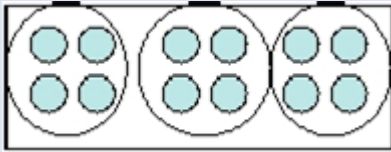
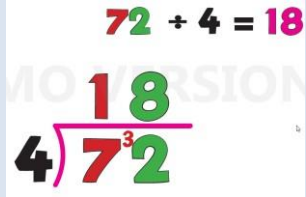
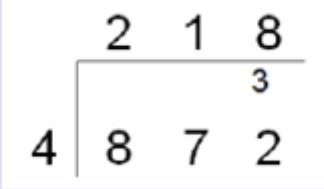
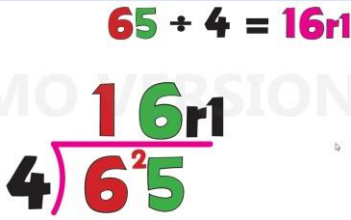
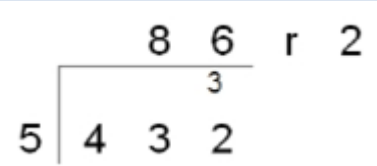
$$\begin{array}{r} 38 \\ \times 27 \\ \hline 266 \quad (38 \times 7) \\ 760 \quad (38 \times 20) \\ \hline 1026 \\ 1 \end{array}$$

Division

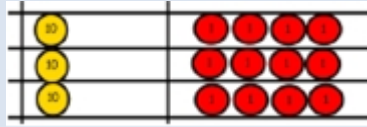
Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 1</p> <p>Yr R</p>	<p>Halving</p>	<p>Half</p> <p>Halve</p> <p>Count out</p> <p>Share out</p> <p>Left</p> <p>Left over</p> <p>...is the same as</p> <p>Equal</p>		<p>One sweet for you, one for me...</p> <p>Is it fair? How many do we each have?</p> 	
<p>Stage 2</p> <p>Yr R</p> <p>Yr 1</p>	<p>Sharing objects</p> <p>Equally</p>	<p>Share</p> <p>Group</p> <p>Divide</p> <p>Half</p> <p>Halve</p> <p>Count out</p> <p>Share out</p> <p>Left</p> <p>Left over</p> <p>Is the same as</p> <p>Is equal to</p>	<p>I have 10 cubes; can you share them equally into 2 groups?</p>  <p>15 shared between 5 is 3:</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>$8 \div 2 = 4$</p> <p>How many groups of 4 are there in 12 stars?</p> 	<p>Share 9 buns between three People:</p> <p>$9 \div 3 = 3$</p>

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 3</p> <p>Yr 1 Yr 2</p>	<p>Division as grouping</p>	<p>Equal groups</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> <p>There are 10 sweets. How many people can have 2 sweets each?</p>  	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p> 	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>
<p>Stage 4</p> <p>Yr 2 Yr 3 Yr 4</p>	<p>Division within arrays</p>	<p>Array Inverse</p>	<p>Link division to multiplication by creating an array and thinking about the number sentences that can be created:</p>  <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	 <p>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</p>	<p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$</p>

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage <u>5</u></p> <p>Yr 3 Yr 4</p>	<p>Division with a remainder</p>	<p>Remainder Equal jumps</p>	<p>$14 \div 3 =$</p> <p>Divide objects into groups or share equally and see how much is left over:</p> 	<p>Draw dots and group them to divide an amount and clearly show a remainder:</p>  <p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p> <p>$13 \div 4 = 3 \text{ r}1$</p>  <p>As knowledge of place value improves, children can begin to jump in multiples of 10:</p> <p>$63 \div 2 = 30 \text{ r}3$</p> 	<p>Children use knowledge of times table facts to quickly calculate divisions involving remainders.</p> <p>For example:</p> $27 \div 5 = 5 \text{ r}2$ <p>Go on to combining knowledge of times tables with place value to calculate more difficult divisions.</p> <p>For example:</p> $137 \div 4 = 34 \text{ r}1$

Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 6</p> <p>Yr3 (2 digit by 1 digit)</p> <p>Yr4 (up to 3 digits by 1 digit)</p> <p>Yr5 (up to 4 digits by a 1 digit remainder. Interpret remainders based on context)</p>	Short division	Bus stop method	<p>Use place value counters to divide using the bus stop method alongside:</p>  <p>$42 \div 3 =$</p> <p>Start with the biggest place value; we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over:</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups:</p> 	<p>Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder:</p>  <p>$872 \div 4 = 218$:</p>  <p>Move onto divisions with a remainder:</p>  <p>$432 \div 5 = 86 \text{ r} 2$:</p> 

We look at how much is in 1 group so the answer is 14:



Finally move into decimal places to divide the total accurately.

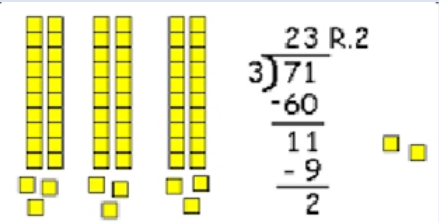


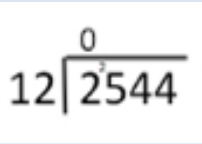
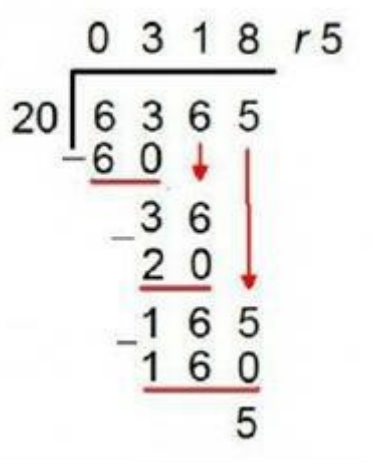
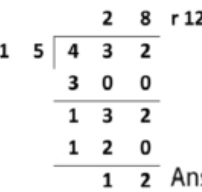
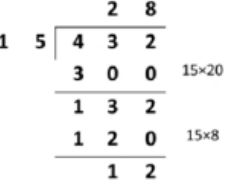
$$511 \div 35 = 14.6:$$

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 16 \\ \underline{14} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

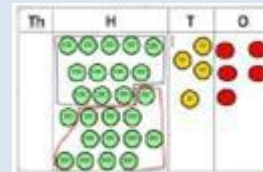
$$87.5 \div 7 = 12.5$$

DEMONSTRATION

$$\begin{array}{r} 12.5 \\ 7 \overline{) 87.5} \\ \underline{7} \\ 17 \\ \underline{14} \\ 35 \\ \underline{35} \\ 0 \end{array}$$

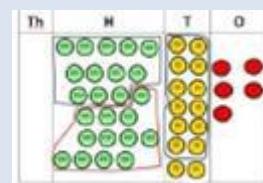
Yr/ Stage	Strategy/ Method	New Vocabulary for the Stage	Concrete	Pictorial	Abstract
<p>Stage 7</p> <p>Yr6 (up to 4 digits by a 2 digit remainder. Interpret remainder as whole numbers, fractions or round)</p>	<p>Long division</p>		<p>It is recommended that instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books. →</p> <p>If needed: $71 \div 3 =$</p> <p>Using Base 10 or place value counters, we start with 7 tens and 1 one, to be divided into 3 groups. We can put 2 tens in each group, so we write a 2 in the ten's column. In all, we've put 6 tens into the groups (3 x 2 tens), so we write 6 tens (60) below.</p> <p>We are left with 11 (1 ten and 1 one). We will need to exchange the ten for 10 ones so we can put 3 ones in each group (using 9 ones in all), and we will have a remainder of 2.</p> 	<p>Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process:</p> <p>Eg. $2544 \div 12$</p> <p>How many groups of 12 thousands do we have? None</p>  <p>Exchange 2 thousand for 20 hundreds:</p>   <p>How many groups of 12 are in 25 hundreds? 2 groups. Circle them.</p>	 <p>$432 \div 15$ becomes</p>  <p>Answer: 28 remainder 12</p> <p>$432 \div 15$ becomes</p>  <p>Answer: $28 \frac{4}{5}$</p> <p>$\frac{12}{15} = \frac{4}{5}$</p>

We have grouped 24 hundreds so can take them off and we are left with one:



$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2:



$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

432 ÷ 15 becomes

$$\begin{array}{r} 28 \cdot 8 \\ 15 \overline{) 432 \cdot 0} \\ \underline{30} \quad \downarrow \\ 132 \quad \downarrow \\ \underline{120} \quad \downarrow \\ 120 \\ \underline{120} \\ 0 \end{array}$$

Answer: 28·8